Innovation in Estimation of Revenue and Cost Functions in PMP Using FADN Information at Regional Level

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Abstract— **The objective of this paper is to present an evolution of PMP model suitable to estimate the revenue function and to provide price elasticity due to the variation of subsidies at farm level, especially if they are decoupled. This problem arises when individual data of farm households in a given region, coming from FADN, are used for implement PMP models finalized to policy analysis. This paper presents the theoretical background of the proposed innovations and empirical evidence on the basis of a sample of farms included in FADN database in Italy.**

Keywords— **Positive mathematical programming, Demand function, Agricultural policies evaluation.**

I. INTRODUCTION

Many papers on PMP applications have considered the PMP methodology suitable for analysing one "single" farm or farm typology. In this case, the approach is called the "standard" approach that considers the cost matrix with elements only in the diagonal according to the very early applications of Positive Mathematical Programming (Howitt, 1995) and Positive Quadratic Programming (Paris and Arfini, 1995).

The adoption of PMP for one single farm, using such a stylized cost matrix, was updated in the 1998 (Paris and Howitt) and in the 2000 (Paris and Arfini) where many farms of the same sample are considered, and the micro information's collected from those farms is used in order to define a stochastic model with frontier farm (Paris and Arfini, 2000). This approach solves the "self selection" problem for each farmer of the sample: how to reproduce exactly the observed land use on the basis of the economic convenience faced by each farmer.

The adoption of the whole sample, in comparison of one single farm, gives the researchers several advantages: a) the possibility to consider all the activities present at farm level; b) to know the variable cost for all the activities present in the sample; c) to give the possibility to all the farms of the sample to diversify their land use according the economic convenience of the crops even if they are not considered in the observed situation.

The estimation a full, positive semidefinite cost matrix was made possible by adopting the Cholesky decomposition using two different approaches: the Maximum Entropy or the Least Square Estimator (Howitt and Paris, 1998). The introduction of an econometric estimation of the total variable cost matrix for all the farms considered in the sample, no matter how large (Howitt and Paris, 1998; Paris and Arfini, 2000), has open a new frontier of research where Mathematical programming is integrated with econometrics. More precisely, its role is to provide estimation starting from the output of mathematical programming models or from the same inputs available for both the methodologies, with the final results to increase the level of analysis of the integrated methodology (Heckelei, 2005).

The possibility of using many farms of a given cross section sample in mathematical model, integrated by econometric estimation, has open to new side of research as the estimation of the revenue function (or demand function) is able to provide price elasticities due to the variation of subsidies (in typology and quantity) at farm level, especially if they are decoupled.

The objective of this paper is to present a new quantitative approach based on PMP, for evaluating the effect of CAP on the agricultural supply dynamics and on the market price modifications, when cross section data are used. This new model is designed for responding to specific demand of policy makers on the issues related to the impact of CAP measures with respect to land allocation, production levels, price variations and farm revenue modifications.

This work is articulated as follows: the first section focuses on the estimation of the PMP approach proposed in this paper, where the calibration of the model is obtained considering also the information about the farm level demand functions for agricultural products that characterizes the given group of farms; the second section concerns an application of the model on a group of farms collected from the IACS database and integrated by FADN information; and the last section concludes with some remarks.

II. REVENUE AND COST FUNCTIONS IN PMP MODEL

The PMP approach presented by Howitt and Paris (1995, 1998) presents an objective function that maximizes, at the last PMP stage, a farm gross margin that takes into account the explicit variable accounting costs of the inputs used inside the production process, but also the part of variable costs that is connected with the farmers' knowledge about their own farm system. In this perspective, the maximized gross margin can be considered the "economic" gross margin, instead of the accounting definition of this term.

However, all PMP models developed according with the above statement explore the supply side of the agricultural sector while avoiding to implement an evaluation of the demand side, by measuring the effects on the output market prices. Indeed, the literature about the PMP models application seems to indicate that such class of models are just developed for investigating the supply side of the agricultural sector, delegating the demand issues side to well-posed problems solved by econometric techniques.

Starting from the above considerations, the methodology proposed in this paper considers the problem of estimating the farm level demand functions associated with a group of farms selected for a policy scenarios evaluation inside a PMP framework. More specifically, the approach is articulated in four phases: 1) cross-section estimation of farm level demand functions using individual data; 2) recovering of the differential marginal costs that lead farmers to choose the observed production plan, considering inside the objective function a non-linear revenue function; 3) estimation of a quadratic cost function; 4) calibration of the base observed situation (the observed production plan) maximizing an objective function composed of the non-linear revenue function estimated in the first phase and the non-linear cost function derived in the third phase.

A. Phase I - Estimation of farm level demand functions

We consider an agricultural region with many entrepreneurs who face a set of aggregate farm-level demand functions for their commodities.

These demand functions assume the following linear form:

$$\mathbf{p} = \mathbf{d} - \boldsymbol{D}\mathbf{x} \tag{1}$$

or, in a sample formulation

$$p_{n,j} = d_j - \sum_{j'=1}^{J'} D_{j,j'} x_{n,j} + v_{n,j}$$

where p, d and x are vectors with dimensions (Jx1) and D a matrix with dimension (JxJ); p, d and x are the vectors of agricultural product prices, the vector of intercepts of demand function and the vector of production quantities, respectively; D is a symmetric positive semidefinite matrix of quantity slopes. J (j=1,…,J) is the number of agricultural processes.

Economic theory assumes that market prices paid to producers vary in relation with the aggregated demand function. Under this assumption, a set of demand functions can be estimated on the basis of a sample of N farms. The term $v_{n,j}$ in (1) represents the deviation of the n-th farm from the regional j-th demand function. If the sample of farms concerns a given geographical region or a sector, it is possible to estimate a set of demand functions for the agricultural products of such a region or a sector. The objective is thus to obtain the set of demand functions (1) using the information of a sample of individual farms.

The relevant information required for estimating (1), consists of prices paid for selling the farm products at the farm level and of output quantities introduced into market. Both types of information are generally available inside the most used agricultural database, as FADN. The methods of estimation vary from generalized least squares, to maximum likelihood, to maximum entropy (ME), etc. In this work, we choose the maximum entropy approach to estimate a well-posed problem. Furthermore, the choice of ME[1] is related to our empirical experience demonstrating that a maximum entropy estimator seems to obtain parameters that provide very realistic results in a simulation phase[2].

The estimation carried out in the present section consists in recovering the demand functions (1) governing the output markets of a sample of 50 farms. The first group of parameters to estimate belongs to the intercept $\mathbf{d}$, while the second group is related to the matrix D. According to the generalized maximum entropy theory of Golan, Judge and Miller, each parameter to recover is equal to the product between a set of probabilities and a set of support values. The objective of the problem is to identify the probability distribution that maximizes the maximum entropy function. The support values are chosen by the researcher[3].

Thus, the intercept can be written as:

$$\mathbf{d}_j = \sum_{p=1}^{P} \mathbf{zd}_{j,p} \mathbf{pd}_{j,p} \qquad (2)$$

where, $\mathbf{zd}_{j,p}$ is the vector of support values, while $\mathbf{pd}_{j,p}$ is the vector of the p (p=1,...,P) probabilities.

We assume that the matrix D is symmetric, positive semidefinite. The simplest and most efficient way to respect those properties is to decompose the matrix D in three components according to the Cholesky factorization method (Paris and Howitt, 1998). On the basis of this method the matrix D is divided in three matrices as follows:

$$\boldsymbol{D} = \boldsymbol{LHL}' \qquad (3)$$

where, D is equal to the product among a unit lower triangular matrix L, a non-negative diagonal matrix H and the transposed of L. The decomposition guarantee in every cases to obtain a symmetric, positive and semidefinite matrix. This same decomposition can be rewritten in a more compact form, so that:

$$\boldsymbol{D} = \boldsymbol{LHL}' = \boldsymbol{RR}' \qquad (4)$$

where the matrix $\boldsymbol{R} = \boldsymbol{LH}^{1/2}$.

In order to estimate the parameters of L and H, it is required to specify a suitable set of support values to associate to an unknown probability distribution, as presented in the following equations:

$$L_{j,j'} = \sum_{p=1}^{P} Zl_{j,j',p} Pl_{j,j',p} \qquad \forall \quad j \neq j' \qquad (5)$$

$$H_{j,j'} = \sum_{p=1}^{P} Zh_{j,j',p} Ph_{j,j',p} \qquad \forall \quad j = j' \qquad (6)$$

Equation (5) states the relation about the unitary triangular matrix $L_{j,j'}$ and the product between the matrix of support values $Zl_{j,j',p}$ and the matrix of probability distribution $Pl_{j,j',p}$. The matrix L is a triangular matrix with unitary values on the diagonal and null values above the diagonal. In equation (6), the matrix $H_{j,j'}$ is equal to the product of the support values $Zh_{j,j',p}$ and the unknown matrix of probability distribution $Ph_{j,j',p}$. H is a non-negative diagonal matrix with null values outside the diagonal.

Keeping into account the statements above, the maximum entropy problem that recovers the demand function (1) starting from a cross-section panel of individual farms is presented below:

$$\begin{aligned} &\max_{p(\cdot)} Hd(p) = \\ &-\sum_{j=1}^{J}\sum_{p=1}^{P} pd_{j,p} \log pd_{j,p} - \sum_{j=1}^{J}\sum_{j'=1}^{J'}\sum_{p=1}^{P} Pl_{j,j',p} \log Pl_{j,j',p} \qquad (7) \\ &-\sum_{j=1}^{J}\sum_{j'=1}^{J'}\sum_{p=1}^{P} Ph_{j,j',p} \log Ph_{j,j',p} - \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{p=1}^{P} pe_{n,j,p} \log pe_{n,j,p} \end{aligned}$$

Subject to:

1. After the publishing of the famous book of Golan, Judge and Miller (1996), the maximum entropy approach has known a new interest among agricultural economists. The idea is to use a physical concept applied to communication technology by Shannon (1948) and in economics by Jaynes (1957) in order to derive parameters when the information is poor and where the traditional econometric techniques prefer not to intervene. For a complete review of maximum entropy theory see Fang et al. (1998). For a detailed discussion about the maximum entropy estimator applied to economics see the book of Golan, Judge and Miller (1996), the paper of Paris and Howitt (1998), Heckelei and Britz (2000), Lansink (1998), Léon et alt. (1999), Lence and Miller (1998).
2. The results achieved applying the ME estimator confirm the important role of this estimator in other fields of the applied sciences (Paris and Howitt, 1998; Shannon, 1948).
3. One of the main criticism addressed to the maximum entropy methods concerns the choice of support values that are submitted to the subjective decision of the researcher (Lansink, 1997).

$$pr_{n,j} = \sum_{p=1}^{P} ze_{n,j,p} pe_{n,j,p} + \\ + \sum_{p=1}^{P} zd_{j,p} pd_{j,p} - \sum_{k=1}^{K} \sum_{j'=1}^{J'} R_{j,j'} R_{k,j'} \overline{x}_{n,k} \ , \forall n \forall j \quad (8)$$

$$R_{j,j'} = \left(\sum_{k=1}^{K} \sum_{p=1}^{P} Zl_{j,k,pp} Pl_{j,k,pp} \right) \\ \left(\sum_{k=1}^{K} \sum_{p=1}^{P} Zd_{j,k,pp} Pd_{j,k,pp} \right)^{1/2} , \forall j \forall j' \quad (9)$$

$$0 = \sum_{n=1}^{N} \sum_{p=1}^{P} ze_{n,j,p} pe_{n,j,p} \quad , \forall j \quad (10)$$

$$\begin{cases} 1 = \sum_{p=1}^{P} pd_{j,p} & , \forall j \\ 1 = \sum_{p=1}^{P} Pl_{j,j',p} & , \forall j \neq j' \\ 1 = \sum_{p=1}^{P} Pd_{j,j',p} & , \forall j = j' \\ 1 = \sum_{p=1}^{P} pe_{n,j,p} & , \forall n \forall j \end{cases} \quad (11)$$

The entropic objective function of problem (7) is maximized with respect to the unknown probability distributions associated with the support values identified by the researcher. Equation (8) states that the observed prices $pr_{n,j}$ are equal to unique demand function plus a farm deviation, $ze_{n,j,p} pe_{n,j,p}$, that measures the distances between n-th observed farm price and the common/regional demand function. Equation (9) performs the Cholesky's decomposition rule established inside the relation (4). The constraint (10) concerns the summation to zero of the farm deviations and the set of constraints (11) state the adding-up relations for the probability distributions. This problem estimates the demand functions of the agricultural market generating the output prices of each farm.

B. Phase II - Recovering of differential marginal costs

The second phase of PMP is devoted to estimating the marginal costs borne by farmers in their input allocation process. When information about accounting variable costs is available, the estimation deals with the differential amount leading to a true economic marginal cost.

The novelty of the proposed PMP approach consists in defining an objective function that depends on the set of farm level demand functions estimated in phase I.

This revenue functions is derived integrating the demand function with respect the output levels, so:

$$\int_0^{\overline{x}} \left(\mathbf{d} - \boldsymbol{D}\mathbf{x} \right) dx = \mathbf{d}\overline{\mathbf{x}} - \frac{1}{2} \overline{\mathbf{x}} \boldsymbol{D} \overline{\mathbf{x}}' \quad (12)$$

The maximization problem of this phase II is usually improperly called as PMP calibration phase. In reality, this stage needs for calibrating the base situation through the differential marginal costs hidden inside the observed production quantities. The objective of this phase is to maximize a non-linear gross margin function subject to typical farm structural constraints (i.e. land) and to calibrating constraints that force the model to reproduce the observed production plan. In algebraic terms, the problem for the n-th farm is written as follows:

$$\max_x GM_0(x) = \\ \sum_{j=1}^{J} \sum_{j'=1}^{J'} \hat{v}_{n,j} x_{n,j} + \hat{d}_j x_{n,j} - \frac{1}{2} x_{n,j} \hat{D}_{j,j'} x_{n,j'} - c_{n,j} x_{n,j} \quad (13)$$

subject to:

$$\sum_{j=1}^{J} A_{n,j,i} x_{n,j} \leq b_{n,i} \quad , \forall i \quad \left[y_{n,i} \right] \quad (14)$$

$$x_{n,j} \leq \overline{x}_{n,j} + \varepsilon \quad , \forall j \quad \left[\lambda_{n,j} \right] \quad (15)$$

$$x_{n,j} \geq 0 \quad , \forall j \quad \left[\mu_{n,j} \right] \quad (16)$$

where $\hat{v}_j$ is the deviation of each farm process from the demand function estimated on the sample of farms. The vectors of deviations is obtained by the previous phase as:

$$\mathbf{v}_{n,j} = \mathbf{ze}_{n,j} \overline{\mathbf{pe}}_{n,j} \quad (17)$$

c_{nj} is the explicit accounting variable cost associated with each output unit at n-th farm level; while $A_{n,j,i}$ and $b_{n,i}$ are respectively the matrix of technology, that is the matrix with the coefficients of input use for obtaining one unit of product, and the vector of input farm capacity i (i.e. land acreage), for i=1,…,I. The coefficients $\hat{d}_j$ and $\hat{D}_{j,j'}$ are the estimates of the corresponding parameters obtained in phase I.

Problem (13)-(16) is optimized when the difference between total revenue and total variable cost is maximized with respect the level of output x. The solution of this problem is known before solving it, because the calibrating constraint (15) imposes that each variable x cannot exceed the observed level of those outputs $\bar{x}$ plus a terms very small ε[4]. The tautological problem (13)-(16) leads to obtain the dual information linked to the calibrating constraint (15), that is λ_j. λ_j is the differential costs to add to the accounting marginal costs c_j in order to obtain a total marginal cost needed for estimating the non-linear cost function of the third phase.

C. Phase III - Non-linear cost function estimation

The objective of the third phase is to estimate the farm cost function starting from the vector of marginal costs estimated in phase II using the shadow prices associated with the calibration constraints. The chosen functional form of the cost function is:

$$C(x) = (\boldsymbol{\lambda} + \mathbf{c})\bar{\mathbf{x}} = \boldsymbol{\alpha}\bar{\mathbf{x}} + \frac{1}{2}\bar{\mathbf{x}}'\boldsymbol{Q}\bar{\mathbf{x}} \qquad (18)$$

where λ and c are, respectively, the vector of the dual values identified in the previous phase and the vector of the farm accounting costs, $\bar{x}$ is the vector of the known production levels and $\boldsymbol{Q}$ the matrix of the non-linear cost function. $\boldsymbol{\alpha}$ is the vector of intercepts for the marginal cost associated to farms processes. In (18) the elements for matrix $\boldsymbol{Q}$ are still unknown and must be obtained through suitable estimation methods. In the literature (see Paris et al., 2000) estimation of cost function through application of the principle maximum entropy is preferred. On the basis of these concepts and the arrangement given by Paris and Howitt (1998), the parameters of vector $\boldsymbol{\alpha}$ and matrix $\boldsymbol{Q}$ can be recovered by maximizing the probability distribution associated with an interval of specified support values. The non linear program of maximum entropy is presented here in the form derived by Cholesky's decomposition according to which the matrix $\boldsymbol{Q} = \boldsymbol{\Gamma W \Gamma}' = \boldsymbol{TT}'$, where $\boldsymbol{\Gamma}$ is a triangular matrix, W a diagonal matrix and $\boldsymbol{T} = \boldsymbol{\Gamma W}^{1/2}$. The problem can then be solved by maximizing a probability distribution for which we know the expected value, which corresponds to the marginal cost $(\boldsymbol{\lambda} + \mathbf{c})$ determined in the second phase. The objective function of the problem of maximum entropy is thus presented as follows:

$$\max_{p(\bullet)} Hc(p) =$$
$$-\sum_{j=1}^{J}\sum_{p=1}^{P} p\alpha_{j,p} \log p\alpha_{j,p} - \sum_{j=1}^{J}\sum_{j'=1}^{J'}\sum_{p=1}^{P} P\varphi_{j,j',p} \log P\varphi_{j,j',p}$$
$$-\sum_{j=1}^{J}\sum_{j'=1}^{J'}\sum_{p=1}^{P} Pw_{j,j',p} \log Pw_{j,j',p} - \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{p=1}^{P} pu_{n,j,p} \log pu_{n,j,p} \qquad (19)$$

where $p\alpha_{j,p}$ are the unknown probability distributions of the intercepts of the cost function, $p\varphi_{j,j',p}$ and $pw_{j,j',p}$ are the probability of the distribution associated with elements of the triangular matrix $\boldsymbol{\Gamma}$ and of the diagonal matrix W respectively. $pu_{n,j,p}$ are elements of the probability of errors. The objective function (19) is maximized considering the information about the process marginal costs at farm level, as follows:

For $x > 0$ at farm level:

$$\lambda_{n,j} + c_{n,j} = \sum_{p=1}^{P} p\alpha_{j,p} z\alpha_{j,p} + \sum_{j'=1}^{J'}\left\{\sum_{k=1}^{K}\left(T_{j,k}T_{k,j'}\right)\right\}\bar{x}_k + \sum_{p=1}^{P} pu_{n,j,p}\, zu_{n,j,w} \quad ,\forall n \forall j \qquad (20)$$

For x not activated at farm level:

$$\lambda_{n,j} + c_{n,j} \le \sum_{p=1}^{P} p\alpha_{j,p} z\alpha_{j,p} + \sum_{j'=1}^{J'}\left\{\sum_{k=1}^{K}\left(T_{j,k}T_{k,j'}\right)\right\}\bar{x}_k + \sum_{p=1}^{P} pu_{n,j,p}\, zu_{n,j,w} \quad ,\forall n \forall j \qquad (21)$$

The equations (20-21) state that the total marginal

4. The meaning of ε is to avoid the linear dependency between the structural constraint and calibrating constraint. For a deeper explanation about the role of e see Howitt (1995), Paris and Howitt (1998) and Gohin and Chantreuil (2000).

cost $(\boldsymbol{\lambda}_{(\cdot)} + \mathbf{c}_{(\cdot)})$ is equal/less or equal to a new marginal cost function common for all the farms sample plus a farm error. $\boldsymbol{T}_{(\cdot)}$ is an element of the matrix T obtained through Cholesky's decomposition. In fact:

$$T_{j,j'} = \sum_{j'=1}^{J} \left\{ \sum_{p=1}^{P} \left(p\varphi_{j,j',w} z\varphi_{j,j',w} \right) \sum_{p=1}^{P} \left(pw_{j,j',p} zw_{j,j',p} \right)^{1/2} \right\} \quad (22)$$

The relations inserted in (22) clarify the role of the support values in the process of estimating the cost matrix. The components $z\varphi_{(\cdot)}$ and $zw_{(\cdot)}$ are the appropriately selected support values (Paris and Howitt, 1998). Associated with the distribution of probability, $p\varphi_{(\cdot)}$ and $zw_{(\cdot)}$, they define the elements of the triangular matrix $\boldsymbol{\Gamma}$ and of the diagonal matrix W. It must be pointed out that the matrix Q is unique and is derived from the marginal costs.

In order to impose that the distribution of deviations is normal, the following adding-up equation is considered:

$$\sum_{n=1}^{N} \sum_{p=1}^{P} pu_{n,j,p} zu_{n,j,p} = 0 \text{ , } \forall j \quad (23)$$

All the probability distributions referred to above must meet the following condition:

$$\begin{cases} \sum_{p=1}^{P} p\alpha_{(\cdot)} = 1 \\ \sum_{p=1}^{P} p\varphi_{(\cdot)} = 1 \\ \sum_{p=1}^{P} pw_{(\cdot)} = 1 \\ \sum_{p=1}^{P} pu_{(\cdot)} = 1 \end{cases} \quad (24)$$

Problem (19)-(24) provides the probability distribution values for the elements of the triangular matrix $\boldsymbol{\Gamma}$, the diagonal matrix W and for the vector of the residual marginal variable costs for each farm in the sample. The cost function specified according to the above method preserves the technical information regarding the calibration constraints.

D. Phase IV - Calibrating observed situation

Finally, after having estimated the revenue and cost functions, we can develop a problem very similar to those in the second phase of the procedure, where a new cost function is inserted and the calibrating constraints are not considered. The problem can be build as follows:

$$\max_{x} GM_1(x) =$$

$$\sum_{j=1}^{J} \sum_{j'=1}^{J'} \left\{ \hat{v}_{n,j} x_{n,j} + \hat{v}_j x_{n,j} - \frac{1}{2} x_{n,j} \hat{D}_{j,j'} x_{n,j'} \right\} \quad (25)$$

$$- \sum_{j=1}^{J} \sum_{j'=1}^{J'} \left\{ \hat{u}_{n,j} x_{n,j} + \hat{\alpha}_j x_{n,j} + \frac{1}{2} x_{n,j} \hat{Q}_{j,j'} x_{n,j'} \right\}$$

subject to:

$$\sum_{j=1}^{J} A_{n,j,i} x_{n,j} \le b_{n,i} \quad , \forall i \quad \left[y_{n,i} \right] \quad (26)$$

$$x_{n,j} \ge 0 \quad , \forall j \quad \left[\mu_{n,j} \right] \quad (27)$$

The error terms $\hat{v}_j$ and $\hat{u}_j$ are derived from the first and third phase of the procedure respectively, and they are specific to each farm. In other terms, they measure the distance between the prices and the costs observed at n-th farm level and the prices and costs estimated for the region considered by the analyst.

Inside the objective function (25) the new quadratic cost function takes the place of the calibrating constraints, establishing the economic bound for the activity allocation choice. In other terms, the latent decision variables revealed in the second phase enter inside the objective function (25) providing an economic calibrating constraint instead of a technical constraint such as the equation (26). The gross margin maximized in (25) is less than the gross margin specified in (23), $GM_0 < GM_1$, because the GM_1 also integrates the dual values associated to the farm activities. For this reason, we can say that the objective function (25) should be considered an economic profit in the sense of the economic theory.

The problem (25)-(27) permits to exactly reproduce the base situation without specific calibrating constraints. Furthermore, applying policy scenario simulations, the non-linear revenue function provide information on the likely variation in agricultural product prices in relation with changes in production levels.

III. EMPIRICAL EVIDENCES

The methodology presented in the previous sections is applied to a sample of farms belonging to the Emilia-Romagna region. The sample is composed by 50 farms placed in the provinces of Parma, Reggio-Emilia, Modena and Bologna and it is extracted from the IACS database. The IACS information, concerning the crop area of each farm, is completed with information deriving from Italian FADN. More specifically, the information concerning the yields, prices and specific variable costs are obtained by the national FADN[5]. 2003 is the reference year. The sample presents a production set of ten crops: cereal mix, alfa-alfa, sugarbeet, durum wheat, fodder crops, maize, barley, silage, soya and soft wheat.

Table 1 Characteristics of the sample

Main information	
Number of farms	50
Incidence of cerelas (in %)	64.5
Incidence of oilseeds (in %)	4.9
Incidence of fodder crops (in %)	19.4
Incidence of sugarbeet (in %)	11.2
Revenue by ha (in euros)	2,001
Variable costs by ha (in euros)	1,466

The aim of the analysis is to estimate the entrepreneurs' response of a single farm payment introduced by the EU regulation 1782/2003. More in detail, the integrated PMP approach is applied to a policy scenario that concerns the total decoupling of the COP crops. The reform of sugarbeet support system is not considered.

The reconstruction of the revenue and cost functions provides the PMP methodology with the ability to analyze the supply and demand sides of the given farm sample. The first aspect concerns the changes in land allocation operated by the farms in relation with the decoupling scenario. Table 2 presents the variation of each crop after the decoupling implementation. The separation between payments and quantity of agricultural products seems to lead farms to abandon part of the cereal acreage for investing in fodder crops, oilseeds and sugarbeet.

5. For further details on the method of merging IACS with FADN database, see Arfini et al. (2005).

The variation in land use has consequences on the production levels and, thus, on market prices. This PMP approach is capable of capturing the price signals in relation to the output variations. This is the second relevant aspect of the model: the simulation can provide variations about market prices of each product. From table 2, it is possible to note the negative variation in the hectares of cereals that leads to an increase in market prices for such products. For example, maize -decreases its acreage by about 15% -, while its price –rises by 19%. Similarly, - fodder crops see a strong increasing in the number of hectares (+48%), while prices foresee a dramatical decrease (-40%).

Table 2 PMP simulation results – Land allocation and Prices

Activities†	Land use		Prices	
	Baseline (ha)	Scenario (Var. %)	Baseline (euros/ton)	Scenario (var. %)
SoftWheat	503.9	-16.5	145.4	+8.2
Durum Wheat	10.1	-26.2	204.5	+4.5
Maize	386.3	-14.8	149.6	+18.9
Barley	130.1	-24.5	131.9	+9.6
Cereals Mix	49.6	-34.8	144.1	+4.5
Silage	58.2	-9.9	40.2	+11.4
Soya	86.5	+11.5	231.7	-7.2
Alfalfa	338.4	+0.5	100.9	-9.9
Other fodder	3.7	+48.4	12.4	-39.9
Sugarbeet	197.7	+6.5	43.1	-10.0

† The model considers also the possibility to activate agricultural area submitted to good practices. The model results indicates that around 10% of the agricultural area would be dedicated to such non-productive activity.

The new production plan due to a decoupling scenario has effects on the main farm economic variables. Table 3 presents a situation where the decrease in revenues and costs leads to improve the farm gross margin (+2%). This is due to a much more intensive reduction of the variable costs (-8.8%) that the farm revenues (-5,9%). The farm strategy within decoupling seems addressed to minimize as much as possible the production costs.

Table 3 PMP simulation results – Main economic variables

Economic variables	Baseline (euros/ha)	Scenario (var. %)
Revenues (gsp+subs.)	2,001	-5.9
Costs	1,466	-8.8
Gross Margin	536	+2.1

The responses of the model in term of quantities, prices and their changes depend in large part on the estimated matrices $\hat{\boldsymbol{Q}}$ and $\hat{\boldsymbol{D}}$ (see Appendix), that integrate the information about the degree of substitution and complementarity among activities.

IV. CONCLUSIONS

The paper presents an evolution of the PMP methodology that can be considered as a generalization of the traditional methodology proposed by Howitt and Paris in 1998. Indeed, the model is able to derive both the demand function that characterizes the agricultural market products and the cost function kept in account by farmers during the production plan definition. The unknown parameters of the revenue and cost functions are recovered by adopting the maximum entropy approach. The last calibration phase maximizes the difference between the farm revenue and cost functions derived by a procedure articulated on four phases.

The results achieved by using the PMP model in assessing policy scenarios can give responses on the supply side, providing the likely modification of the land use and the production level, and on the demand side, providing information about the dynamics of prices. The model can respond to policy maker's needs providing in a unique evaluation tool, the information about the demand and supply reactions in relation with changes in agricultural policy measures.

The proposed model estimates the observed situation and provides predictions using economic, strategic and structural information available in the FADN sample. The calibration and the simulations phases are both carried out with respect to the single farm, keeping into account the specific allocation behaviour of each farm and using it for estimating the likely effects of policy measures.

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APPENDIX

REVENUE AND COST FUNCTION MATRICES

Demand function matrix $\hat{D}$

		J'									
		Cereals Mix	Fodder Crops	Sugarbeet	Durum Wheat	Alfalfa	Maize	Barley	Silage	Soya	Soft Wheat
J	Cereals Mix	**0.7296**	0.0378	0.0002	0.0791	0.1206	-0.0742	-0.0402	-0.0028	-0.2774	-0.0195
	FodderCrops		**0.1171**	0.0098	-0.0157	0.0228	-0.0592	-0.0325	-0.0165	0.014	-0.0151
	Sugarbeet			**0.0199**	0.0014	-0.0004	-0.0167	0.011	-0.0039	-0.0126	-0.0158
	DurumWheat				**2.0305**	-0.0244	0.0034	-0.0251	-0.017	-0.1762	0.0032
	Alfalfa					**1.7535**	0.0114	-0.0423	-0.0516	-0.1563	-0.0155
	Maize						**0.2615**	0.0302	-0.0013	-0.0442	-0.075
	Barley							**0.7192**	0.0079	-0.0107	-0.1155
	Silage								**0.0961**	0.0289	0.0028
	Soya									**1.4435**	-0.0061
	SoftWheat										**0.2184**

Cost function matrix $\hat{Q}$

		J'									
		Cereals	Fodder Crops	Sugarbeet	Durum Wheat	Alfalfa	Maize	Barley	Silage	Soya	Soft Wheat
J	Cereals	**0.0526**	4.87E-05	-0.0006	4.96E-06	-5.85E-06	-0.0001	2.74E-08	-0.0002	-4.21E-05	-2.59E-06
	FodderCrops		**0.0043**	-5.57E-07	-4.94E-09	-6.00E-09	-6.16E-08	4.21E-09	-1.60E-07	-4.26E-08	-4.15E-09
	Sugarbeet			**0.0006**	5.44E-08	6.44E-08	7.87E-08	-3.00E-10	1.93E-06	4.63E-07	-2.85E-08
	DurumWheat				**0.5185**	0.0543	-0.0034	0.0282	0.0028	0.0105	0.0012
	Alfalfa					**0.0287**	-0.0004	0.003	0.0003	0.0011	0.0001
	Maize						**0.0072**	-0.0002	-1.84E-05	-0.0001	-7.91E-06
	Barley							**0.0309**	0.0002	0.0006	0.0001
	Silage								**0.0025**	0.0001	6.37E-06
	Soya									**0.1042**	-0.0029
	SoftWheat										**0.0074**